1. Find 2 linearly independent solutions to the differential equation.

\[ y'' + x^2y = 0 \]

2. A vat has a volume of 100 liters. It initially contains 50 liters of pure water. Brine with a concentration of 1 gram per liter begins to flow into the vat at a rate of 2 liters per minute. The mixed solution escapes through a leak at a rate of 1 liter per minute. How much salt is there in the vat when it begins to overflow?

3. Solve the initial value problem

\[ y'' + y' + y = 0, \quad y(0) = 0, y'(0) = 1 \]

4. Is the series convergent? If so find its sum?

\[ \sum_{n=3}^{\infty} \frac{1}{n^2 - n} \]

(a) Suppose \( f \) is a continuous positive strictly decreasing function for \( x \geq 1 \) and \( a_n = f(n) \).

By drawing a picture, rank the following in increasing order:

\[ \int_1^6 f(x) \, dx \quad \sum_{i=1}^{3} a_i \quad \sum_{i=2}^{6} a_i \]

5. Find a particular solution of the differential equation

\[ y'' - y' - y = e^{2x} + 1 \]

6. Evaluate the following integrals:

(a)

\[ \int_1^{\infty} \frac{1}{x^{\sqrt{2}}} \, dx \]

(b)

\[ \int x^3 e^{x^2} \, dx \]